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FIX OF ELECTRODYNAMICS

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ABSTRACT

Lorentz's statement is known that "... Heaviside and Hertz gave a clear and concentrated form to Maxwell's equations." At present, after Heaviside and Hertz, the applicability of Maxwell's equations to all phenomena of electrodynamics and electrical devices is indisputable. But this indisputability applies only to the case when these equations are solved numerically. The analytical solution – the wave equation of electrodynamics, which contradicts the law of conservation of energy and many analytical consequences of Maxwell's equations, an indispensable attribute of which is the vector potential – are very far from reality. This happened because Maxwell's equations, which the author along with many admire, must be correctly resolved. Below is a discussion of what these correct decisions are.

Keywords: Maxwell's equations, electrodynamics, electrical field strengths and magnetic field strengths, ocean square waves.

INTRODUCTION

My criticism of the theory of electromagnetism follows not from logic, not from my own postulates, not from the notorious physical meaning but from mathematics. The author of this paper has been publishing mathematically proven changes in some provisions of electrodynamics for 10 years in Russian and English in the public domain and the author is extremely surprised at the lack of a keen interest in fundamentally new solutions: The author would like to hear opinions on agreement or refutation. That is why the author has written this article.

The author of this paper will not refer to a specific author, so as not to accuse someone of all the sins of the modern theory of electromagnetism. Let this specific author join the discussion himself/herself (if the specific author wants to). The author of this paper will not, where necessary and not necessary, use vector calculus because (as will be clear from what follows) it has played a cruel joke on electrodynamics: with its help, the reader can easily inadvertently or deliberately get an incorrect result, and sometimes its use simply does not allow the reader to get the real result.

The author of this paper will start from the moment when, according to the well-known statement of Lorentz, "... *Heaviside and Hertz gave a clear and concentrated form to Maxwell's equations*." They are known to have obtained from twelve Maxwell’s equations, four equations in the vector form. Let's write them down as follows:

$$\mathrm{rot}(\mathbf{E}) + \mu\frac{\partial \mathbf{H}}{\partial t} = 0 \tag{1}$$

$$\mathrm{rot}(\mathbf{H}) - \varepsilon\frac{\partial \mathbf{E}}{\partial t} = \mathbf{J} \tag{2}$$

$$\mathrm{div}(\mathbf{E}) = 0 \tag{3}$$

$$\mathrm{div}(\mathbf{H}) = 0 \tag{4}$$

where **E**, **H**, **J** are the vectors of electrical field strengths, magnetic field strengths, and conduction currents; ε, μ are the electric permittivity and the magnetic permeability, respectively. In Cartesian coordinates $\{x, y, z\}$, this set of equations has the following form:

$$\begin{cases} \dfrac{\partial H_z}{\partial y} - \dfrac{\partial H_y}{\partial z} - \varepsilon\dfrac{\partial E_x}{\partial t} - J_x = 0 \\ \dfrac{\partial H_x}{\partial z} - \dfrac{\partial H_z}{\partial x} - \varepsilon\dfrac{\partial E_y}{\partial t} - J_y = 0 \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} - \varepsilon\dfrac{\partial E_z}{\partial t} - J_z = 0 \\ \dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} + \mu\dfrac{\partial H_x}{\partial t} = 0 \\ \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} + \mu\dfrac{\partial H_y}{\partial t} = 0 \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} + \mu\dfrac{\partial H_z}{\partial t} = 0 \\ -\dfrac{\partial E_x}{\partial x} - \dfrac{\partial E_y}{\partial y} - \dfrac{\partial E_z}{\partial z} + \dfrac{\rho}{\varepsilon} = 0 \\ \dfrac{\partial H_x}{\partial x} + \dfrac{\partial H_y}{\partial y} + \dfrac{\partial H_z}{\partial z} - \dfrac{\sigma}{\mu} = 0 \end{cases} \tag{5}$$

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The author notes right away that not a single bad word about these equations will be said further. Moreover, the author has no doubts about the validity of these equations, and it seems that these equations are thrown into our civilization through the minds of their creators, as a concentrated source of knowledge. What is a consequence of these equations turns out to be true in the experiment, and the consequences themselves take on an elegant mathematical form after cumbersome mathematical transformations. Such a result in itself creates confidence in the correctness of the investigation.

So, what's the matter that you want to fix? - the reader will exclaim. The point is that the equations must be correctly resolved.`

The creators themselves introduced some ideas that are not visible in the equations but prevent getting the correct solution of these equations. For instance,

- The wave equation contradicts the law of conservation of energy;
- The flow of energy entering the wire from the outside does not go along and outside the wire;
- Analytical solution of Maxwell's equations is not unique;
- The vector potential used by Maxwell when deriving the equations contradicts these equations;
- Lorentz's magnetic force equation does not complement Maxwell's equations' set but follows from it;
- The minimum principle is not a necessary and sufficient condition.

At present, after Heaviside and Hertz, the applicability of Maxwell's equations to all phenomena of electrodynamics and electrical devices without exception is undeniable (as will be clear from what follows). However, it is not always possible to describe these phenomena and devices in the form of a solution to the complete set of Maxwell's equations, and not some subsets of this set. The author shows in my publications that, by applying the full set of Maxwell's equations, it is possible to find a solution for those experimental conditions when the experiment, it would seem, contradicts these equations.

On the wave equation

Let us consider the simplest case, namely the solution of Maxwell's equations for a vacuum in the absence of the longitudinal strengths and the conduction currents. The wave equation in this case takes the following form:

$$E_x = e_x \cos(ax + by + \chi z + \omega t + \varphi_0) \tag{6}$$
$$E_y = e_y \cos(ax + by + \chi z + \omega t + \varphi_0) \tag{7}$$
$$H_x = h_x \cos(ax + by + \chi z + \omega t + \varphi_0) \tag{8}$$
$$H_y = h_y \cos(ax + by + \chi z + \omega t + \varphi_0) \tag{9}$$

where e_x, e_y, h_x, h_y, a, b, χ, ω, φ_0 are some constants. Functions (7)-(9) are related by equations (1)-(5) and, in particular, by equations of the following form:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \varepsilon \frac{\partial E_x}{\partial t} = 0 \tag{10}$$
$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \varepsilon \frac{\partial E_y}{\partial t} = 0 \tag{11}$$

which establish a connection between the constants e_x, e_y, h_x, h_y. According to the indicated equations, it is possible to construct some graphs of functions (6)-(9) that is shown in Figure 1. The sinusoids propagate along the z-axis. The functions E_x, H_x oscillate along the blue x-axis, and the functions E_y, H_y oscillate along the red y-axis.

It can be seen that in this case, the functions **H** coincide in phase with the functions **E**. This means that there are points on the z-axis where all the strengths are equal to zero. At these points, the wave energy is zero. Therefore, there is no energy at these points, i.e. in such a solution, the law of conservation of energy is not always satisfied, and this contradicts the very spirit of this law. The author did not discover an America because it is well-known that the law of conservation of energy is observed on average. But electrodynamics pretends that everything is fine.

So, the well-known solution of Maxwell's equations in the form of a wave equation is not acceptable because in such a solution the law of conservation of energy is satisfied only on average. In Khmelnik (2021a), a solution to Maxwell's equations is proposed that does not have this shortcoming. In this case, it looks like this one:

$$E_x = e_x(r)\sin\big((\alpha + 1)\varphi + \chi z + \omega t\big) \tag{12}$$
$$E_y = e_y(r)\cos\big((\alpha - 1)\varphi + \chi z + \omega t\big) \tag{13}$$
$$H_x = h_x(r)\cos\big((\alpha + 1)\varphi + \chi z + \omega t\big) \tag{14}$$
$$H_y = h_y(r)\sin\big((\alpha - 1)\varphi + \chi z + \omega t\big) \tag{15}$$

where

$$e_x(r) = e_y(r) = Ar^{(\alpha-1)} \tag{16}$$

$$h_x(r) = h_y(r) = -\sqrt{\frac{\varepsilon}{\mu}} e_r(r) \tag{17}$$

$$\chi = \omega\sqrt{\mu\varepsilon} \tag{18}$$
$$r = \sqrt{x^2 + y^2} \tag{19}$$
$$\varphi = \mathrm{arctg}(y/x) \tag{20}$$

with A, α, ω being some constants.

Similar to the previous case, consider the graphs of functions (12)-(15) shown in Figure 2 for A = 1, α = 0.5, t

= 0, χ = 1.5. As shown in Figure 1, these sinusoids propagate along the z-axis. The E_x, H_x functions oscillate along the blue x-axis, while the E_y, H_y functions oscillate along the red y-axis. But here the electric field strengths and magnetic field strengths are shifted in phase by a quarter of a period and the electromagnetic energy flux on a cylinder of a given radius retains a certain value throughout the entire z-axis, despite the periodic change in these strengths. The consequences of this simple fact in many respects contradict the existing electrodynamics.

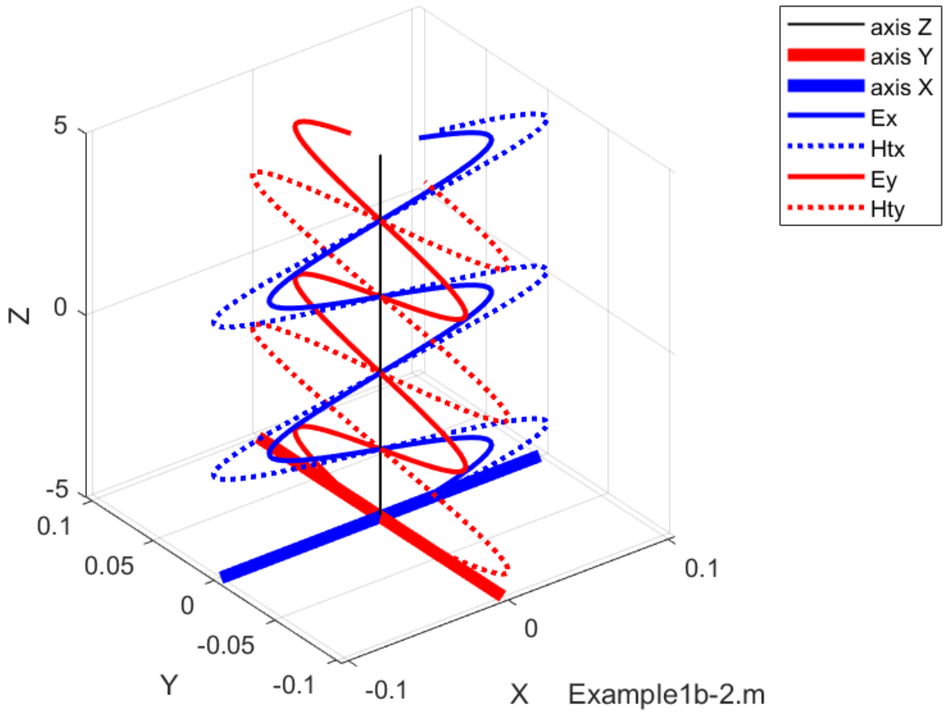

Fig. 1. The visualization of the components of the electrical field strengths (E_x, E_y) and magnetic field strengths (H_x, H_y).

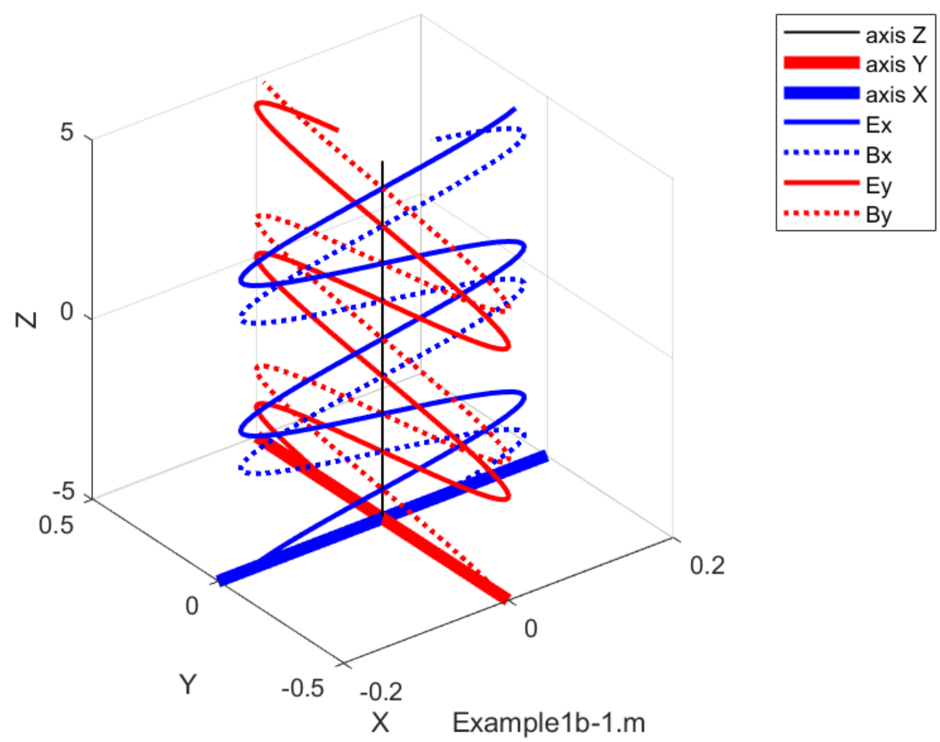

Fig. 2. The visualization of the parameters in functions (12)-(15).

Now the law of conservation of energy in electrodynamics is preserved. Electrodynamics has become the same as all branches of physics.

Solution of Maxwell's equations for a vacuum

Consider this solution in the general case, i.e. with the existence of the longitudinal strengths (Khmelnik, 2021a). In this case compared to the previous one, the solution becomes more complicated and takes the following form (Khmelnik, 2021a):

$$E_x = e_r \mathrm{si} \cdot \cos(\varphi) + e_\varphi \mathrm{co} \cdot \sin(\varphi) \tag{21}$$
$$E_y = e_r \mathrm{si} \cdot \sin(\varphi) + e_\varphi \mathrm{co} \cdot \cos(\varphi) \tag{22}$$
$$E_z = e_z(r)\mathrm{co} \tag{23}$$
$$H_x = h_r \mathrm{co} \cdot \cos(\varphi) + h_\varphi \mathrm{si} \cdot \sin(\varphi) \tag{24}$$
$$H_y = h_r \mathrm{co} \cdot \sin(\varphi) + h_\varphi \mathrm{si} \cdot \cos(\varphi) \tag{25}$$
$$H_z = h_z(r)\mathrm{si} \tag{26}$$

where

$$\mathrm{co} = \cos(\alpha\varphi + \chi z + \omega t) \tag{27}$$
$$\mathrm{si} = \sin(\alpha\varphi + \chi z + \omega t) \tag{28}$$
$$e_z = \mathrm{A} r^{-\alpha} \tag{29}$$
$$e_r = -\frac{1}{2}\left(\frac{\alpha}{\chi r} + \frac{\chi r}{\alpha}\right) e_z \tag{30}$$
$$e_\varphi = \frac{1}{2}\left(\frac{\alpha}{\chi r} - \frac{\chi r}{\alpha}\right) e_z \tag{31}$$
$$h_r = k e_r \tag{32}$$
$$h_\varphi = -k e_\varphi \tag{33}$$
$$h_z = -k e_z \tag{34}$$
$$k = \sqrt{\frac{\varepsilon}{\mu}} \tag{35}$$
$$\chi = \omega\sqrt{\mu\varepsilon} \tag{36}$$
$$r = \sqrt{x^2 + y^2} \tag{37}$$
$$\varphi = \mathrm{arctg}(y/x) \tag{38}$$
$$A, \alpha, \omega = \mathrm{const} \tag{39}$$

Here, as well as in the previous case, the electric and magnetic strengths are shifted in phase by a quarter of the period, and the electromagnetic energy flux on a cylinder of a given radius retains a certain value throughout the entire z-axis, despite the periodic change in these strengths.

On the vector potential

Consider the vector potential **A** in electrodynamics, which satisfies the following equation:

$$\mathrm{rot}(\mathbf{A}) = \mu \mathbf{H} \tag{40}$$

Consider again the solution of Maxwell's equations for a vacuum in the general case but in the cylindrical coordinate system $\{r, \varphi, z\}$, keeping the notation adopted in the previous section. In this case, the solution will take the form:

$$H_r = h_r(r)\mathrm{co} \tag{41}$$
$$H_\varphi = h_\varphi(r)\mathrm{si} \tag{42}$$
$$H_z = h_z(r)\mathrm{si} \tag{43}$$
$$E_r = e_r(r)\mathrm{si} \tag{44}$$
$$E_\varphi = e_\varphi(r)\mathrm{co} \tag{45}$$
$$E_z = e_z(r)\mathrm{co} \tag{46}$$

where

$$\mathrm{co} = \cos(\alpha\varphi + \chi z + \omega t)$$
$$\mathrm{si} = \sin(\alpha\varphi + \chi z + \omega t)$$

In this case, the vector potential components will take the following form:

$$A_r = a_r(r)\mathrm{co} \tag{47}$$
$$A_\varphi = a_\varphi(r)\mathrm{si} \tag{48}$$
$$A_z = a_z(r)\mathrm{si} \tag{49}$$

Let us write down further the divergence equation for the vector **H**:

$$\frac{H_r}{r} + \frac{\partial H_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial H_\varphi}{\partial \varphi} + \frac{\partial H_z}{\partial z} = 0 \tag{50}$$

After substituting equations (41)-(43) into this equation and reducing by the coefficients co and si, we find:

$$\frac{1}{r} h_r + \dot{h}_r + \frac{\alpha}{r} \cdot h_\varphi + \chi h_z = 0 \tag{51}$$

In equation (51) and the corresponding equations written below, dots denote derivatives with respect to r. Equation (40) will take the following form:

$$\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} = \mu H_r \tag{52}$$
$$\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} = \mu H_\varphi \tag{53}$$
$$\frac{A_\varphi}{r} + \frac{\partial A_\varphi}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \varphi} = \mu H_z \tag{54}$$

Substituting equations (41)-(43) and (47)-(49) into equations (52)-(54) and reducing by the coefficients co and si, we find:

$$\frac{1}{r}a_z(r)\alpha - a_\varphi(r)\chi - \mu h_r(r) = 0 \tag{55}$$

$$-a_r(r)\chi - \dot{a}_z(r) - \mu h_\varphi(r) = 0 \tag{56}$$

$$\frac{a_\varphi(r)}{r} + \dot{a}_\varphi(r) + \frac{a_r(r)}{r}\alpha - \mu h_z(r) = 0 \tag{57}$$

From here, first of all, it follows that in the definition of the vector potential according to equations (47)-(49), another distribution of the functions co and si is not allowed, since otherwise, the terms of each equation from equations (55)-(57) will contain various functions co and si that cannot be reduced.

The set of equations (55)-(57) determines the coefficients *a* depending on the known coefficients *h*. From (15) we find:

$$a_r = -\frac{1}{\chi}\left(\dot{a}_z + \mu h_\varphi\right) \tag{58}$$

Combining equations (57) and (58), we find:

$$\frac{1}{r}a_\varphi + \dot{a}_\varphi - \frac{\alpha}{r\chi}\dot{a}_z - \frac{\alpha}{r\chi}\mu h_\varphi - \mu h_z = 0 \tag{59}$$

From (55) we find:

$$a_z = \frac{r}{\alpha}\left(a_\varphi\chi + \mu h_r\right) \tag{60}$$

$$\dot{a}_z = \frac{1}{\alpha}\left(a_\varphi\chi - \mu h_r\right) + \frac{r}{\alpha}\dot{a}_\varphi\chi \tag{61}$$

From equations (59) and (60) we can find that

$$\frac{1}{r}a_\varphi + \dot{a}_\varphi - \mu\left(\frac{\alpha}{r\chi}h_\varphi + h_z\right)$$
$$-\frac{\alpha}{r\chi}\left(\frac{1}{\alpha}\left(a_\varphi\chi - \mu h_r\right) + \frac{r}{\alpha}\dot{a}_\varphi\chi\right) = 0$$

or

$$\frac{1}{r\chi}h_r - \frac{\alpha}{r\chi}h_\varphi - h_z = 0$$

This condition must be met in order for the set of equations (55) and (56) to have a solution. But it contradicts the condition (51). Therefore, the set of equations (55) and (56) is incompatible with the given solution of the set of Maxwell's equations.

A similar conclusion can be drawn for other solutions of the set of Maxwell equations for applications in technical devices. Thus, in the general case, the definition of a vector potential contradicts Maxwell's equations. Note that in the absence of longitudinal strengths, the vector potential exists. When solving in the form of a wave equation, the vector potential exists but the solution itself does not exist.

So, in the general case, the vector potential in electrodynamics does not exist. This means that thousands of books and articles that begin with a reference to vector potential are wrong. But on the other hand, the condition of gauge invariance disappears from electrodynamics, which can be satisfied in many ways, and introduces arbitrariness into physics, which contradicts the very spirit of classical physics.

Solution of Maxwell's equations for a wire with conduction current

Here we will consider a rectilinear cylindrical wire of unlimited length, in which there is a specific electrical conductivity σ. In this case, not only displacement currents $\varepsilon\partial E/\partial t$ but also conduction currents J are present in the wire, and Maxwell's equations take the form of equations (1)-(4).

The conduction current is proportional to the electrical intensity, but in the conductor their phases do not coincide. This means that there is no single solution of Maxwell's equations in which the conduction current is also present. Therefore, we will look for a solution as the sum of two monochromatic solutions with the same frequencies. But the main rationale for this approach is electrical engineering.

Here we consider only the case when there is a specific electrical conductivity σ but $\varepsilon = 0$. Then equations (1)-(4) take the following form:

$$\mathrm{rot}(\mathbf{E}) + \mu\frac{\partial \mathbf{H}}{\partial t} = 0$$
$$\mathrm{rot}(\mathbf{H}) = \mathbf{J}$$
$$\mathrm{div}(\mathbf{E}) = 0$$
$$\mathrm{div}(\mathbf{H}) = 0$$

The solution of this set of equations has the following form (Khmelnik, 2021a):

$$E_x = e_r\mathrm{si}\cdot\cos(\varphi) + e_\varphi\mathrm{co}\cdot\sin(\varphi) \tag{62}$$

$$E_y = e_r\mathrm{si}\cdot\sin(\varphi) + e_\varphi\mathrm{co}\cdot\cos(\varphi) \tag{63}$$

$$E_z = e_z\mathrm{co} \tag{64}$$

$$H_x = h_r\mathrm{co}\cdot\cos(\varphi) + h_\varphi\mathrm{si}\cdot\sin(\varphi) \tag{65}$$

$$H_y = h_r\mathrm{co}\cdot\sin(\varphi) + h_\varphi\mathrm{si}\cdot\cos(\varphi) \tag{66}$$

$$H_z = h_z(r)\mathrm{si} \tag{67}$$

$$J_x = j_r\mathrm{co}\cdot\cos(\varphi) + h_\varphi\mathrm{si}\cdot\sin(\varphi) \tag{68}$$

$$J_y = j_r\mathrm{co}\cdot\sin(\varphi) + h_\varphi\mathrm{si}\cdot\cos(\varphi) \tag{69}$$

$$J_z = j_z(r)\mathrm{si} \quad (70)$$

where

$$\mathrm{co} = \cos(\alpha\varphi + \chi z + \omega t) \quad (71)$$
$$\mathrm{si} = \sin(\alpha\varphi + \chi z + \omega t) \quad (72)$$
$$e_z = \mathrm{A}r^{-\alpha} \quad (73)$$
$$e_\varphi = e_r = Br^{1-\alpha} \quad (74)$$
$$B = \frac{\chi \mathrm{A}}{2(1-\alpha)} \quad (75)$$
$$h_r = ke_r \quad (76)$$
$$h_\varphi = -ke_\varphi \quad (77)$$
$$h_z = -ke_z \quad (78)$$
$$j_r = \sigma e_r \quad (79)$$
$$j_\varphi = \sigma e_\varphi \quad (80)$$
$$j_z = \sigma e_z \quad (81)$$
$$0 < \alpha < 1 \quad (82)$$
$$k = \sqrt{\frac{\sigma}{\mu\omega}} \quad (83)$$
$$\chi = \sqrt{\sigma\mu\omega} \quad (84)$$
$$r = \sqrt{x^2 + y^2} \quad (85)$$
$$\varphi = \mathrm{arctg}(y/x) \quad (86)$$

with A, α, ω being constants.

In this solution, the magnetic field strengths and electrical field strengths are antiphase, the conduction currents are in phase with the magnetic field strengths.

Let us also consider the energy flows in the wire, using the cylindrical coordinates $\{r, \varphi, z\}$. In this case, the strengths are respectively determined as follows:

$$H_r = h_r(r)\mathrm{si} \quad (87)$$
$$H_\varphi = h_\varphi(r)\mathrm{co} \quad (88)$$
$$H_z = h_z(r)\mathrm{co} \quad (89)$$
$$E_r = e_r(r)\mathrm{co} \quad (90)$$
$$E_\varphi = e_\varphi(r)\mathrm{si} \quad (91)$$
$$E_z = e_z(r)\mathrm{si} \quad (92)$$

The electromagnetic energy flux density, namely the Poynting vector is determined by the following formula:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \quad (93)$$

In the cylindrical coordinates, it has three components $\{S_r, S_\varphi, S_z\}$ directed along the radius, along the circumference, along the z-axis, respectively. They are determined by the following formula:

$$\mathbf{S} = \begin{bmatrix} S_r \\ S_\varphi \\ S_z \end{bmatrix} = (\mathbf{E} \times \mathbf{H}) = \begin{bmatrix} E_\varphi H_z - E_z H_\varphi \\ E_z H_r - E_r H_z \\ E_r H_\varphi - E_\varphi H_r \end{bmatrix} \quad (94)$$

or, taking into account the previous formulas,

$$S_r = \eta\left(e_\varphi h_z - e_z h_\varphi\right)\mathrm{co}\cdot\mathrm{si} \quad (95)$$
$$S_\varphi = \eta(e_z h_r \mathrm{co}^2 - e_r h_z \mathrm{si}^2) \quad (96)$$
$$S_z = \eta\left(e_r h_\varphi \mathrm{si}^2 - e_\varphi h_r \mathrm{co}^2\right) \quad (97)$$

Substituting here formulas (76)-(78), we get:

$$S_r = \eta\left(-ke_\varphi e_z + ke_z e_\varphi\right)\mathrm{co}\cdot\mathrm{si} = 0 \quad (98)$$
$$S_\varphi = \eta(ke_z e_r \mathrm{co}^2 + ke_r e_z \mathrm{si}^2) = \eta k e_r e_z \quad (99)$$
$$S_z = \eta\left(-ke_r e_\varphi \mathrm{si}^2 - ke_\varphi e_r \mathrm{co}^2\right) = -\eta k e_r e_\varphi \quad (100)$$

It follows from formula (98) that there is no radial energy flow directed perpendicular to the wire surface, but, as follows from (99) and (100), there are energy flows directed along the wire and around the wire circumference. Both of these flows have a value that is constant in time. They are streams of active energy. The idiotic idea (according to Feynman *et al*. (1964)) that the energy flow comes into the wire from the outside and only to turn into the energy of the thermal motion of electrons is not confirmed.

Variational principle in electrodynamics

The variational principle of least action exists in all branches of physics. It is known that Maxwell's equations are also derived from the principle of least action. For this, the concept of the existence of a vector potential is used, then a certain functional is formulated with respect to such a potential and a scalar electric potential called the action. By varying the action with respect to the vector magnetic potential and the scalar potential, the condition for the minimum of this functional is found. However, it was shown above that the vector potential is not compatible with Maxwell's equations. Therefore, the conclusion under consideration cannot be considered conclusive. We also note that the resulting functional does not include thermal energy losses arising from conduction currents. The matter is further complicated by the fact that in the symmetric form of Maxwell's equations (in the presence of both magnetic and electric charges) the electromagnetic field cannot be described using a vector potential that is continuous throughout space. Therefore, Maxwell's symmetric equations are not derived from the variational principle of least action, even if we assume the existence of a vector potential.

Thus, to derive the Maxwell equations from the variational principle, another functional must be found that does not involve the use of a vector potential and allows one to take into account the energy dissipation. The author proposed the full action extremum principle, which also takes into account heat losses. This principle is described in (Khmelnik, 2014). There is also a functional for which the complete set of symmetric Maxwell equations is a necessary and sufficient condition for the existence of a unique optimum.

In addition, the proposed functional can be used to resolve Maxwell's equations. The fact is that the functional used in this or that principle is an integral. It is possible to construct an algorithm for moving along the surface described by the integrand in the direction of the optimal line. When the optimum is reached, the equations are thus resolved, which are the conditions for the existence of this optimum.

Before formulating the functional as a whole, consider the functional of the following form:

$$\Phi_o = \oint_z \left\{ \oint_y \left\{ \oint_x f(x,y,z)\, dx \right\} dy \right\} dz \tag{101}$$

where

$$\begin{aligned} f(x,y,z) &= H_x \frac{\partial E_z}{\partial y} - H_x \frac{\partial E_y}{\partial z} + H_y \frac{\partial E_x}{\partial z} \\ &- H_y \frac{\partial E_z}{\partial x} + H_z \frac{\partial E_y}{\partial x} + H_z \frac{\partial E_x}{\partial y} \\ &- E_x \frac{\partial H_z}{\partial y} + E_x \frac{\partial H_y}{\partial z} - E_y \frac{\partial H_x}{\partial z} \\ &+ E_y \frac{\partial H_z}{\partial x} - E_z \frac{\partial H_y}{\partial x} + E_z \frac{\partial H_x}{\partial y} \end{aligned} \tag{102}$$

In (Khmelnik, 2014), it is proved that the extremals of this functional are equations of the following form:

$$\mathrm{rot}(\mathbf{H}) = 0 \tag{103}$$
$$\mathrm{rot}(\mathbf{E}) = 0 \tag{104}$$

For the convenience of further presentation, the integrand in (101) will be denoted as $\mathfrak{I}(\mathbf{H}, \mathbf{E})$. In this case, functional (101) takes the following form:

$$\Phi_o = \oint_z \left\{ \oint_y \left\{ \oint_x \{\mathfrak{I}(\mathbf{H},\mathbf{E})\}\, dx \right\} dy \right\} dz \tag{105}$$

It can be seen that

$$\mathfrak{I}(\mathbf{H},\mathbf{E}) = \mathbf{H}\cdot \mathrm{rot}(\mathbf{E}) - \mathbf{E}\cdot \mathrm{rot}(\mathbf{H}) \tag{106}$$

Consider now the following functional:

$$\Phi = \int_{t=0}^{T} \left\{ \int_z \left\{ \int_y \left\{ \int_x (\mathbf{\Phi_1} dx) \right\} dy \right\} dz \right\} dt \tag{107}$$

where

$$\begin{aligned} \mathbf{\Phi_1} &= \frac{1}{2}\{\mathfrak{I}(\mathbf{H}', \mathbf{E}') - \mathfrak{I}(\mathbf{H}'', \mathbf{E}'')\} \\ &+ \frac{\mu}{2}\left\{\mathbf{H}' \frac{d\mathbf{H}''}{dt} - \mathbf{H}'' \frac{d\mathbf{H}'}{dt}\right\} + \frac{\varepsilon}{2}\left\{-\mathbf{E}' \frac{d\mathbf{E}''}{dt} + \mathbf{E}'' \frac{d\mathbf{E}'}{dt}\right\} \\ &+ \left\{-\mathbf{K}' \left(\mathrm{div}(\mathbf{E}') - \frac{\rho}{2\varepsilon}\right) + \mathbf{K}'' \left(\mathrm{div}(\mathbf{E}'') - \frac{\rho}{2\varepsilon}\right)\right\} \\ &+ \left\{\mathbf{L}' \left(\mathrm{div}(\mathbf{H}') - \frac{\sigma}{2\mu}\right) - \mathbf{L}'' \left(\mathrm{div}(\mathbf{H}'') - \frac{\sigma}{2\mu}\right)\right\} \end{aligned} \tag{108}$$

In this functional, all variable functions are represented as sums: $\mathbf{H} = \mathbf{H'} + \mathbf{H''}$, etc. The necessary conditions for the extremum of such a functional as a functional of functions of several independent variables are the Ostrogradsky equations (Elsgoltz, 2000). Applying them and differentiating with respect to the variables $\mathbf{E'}$, $\mathbf{E''}$, $\mathbf{H'}$, $\mathbf{H''}$, $\mathbf{K'}$, $\mathbf{K''}$, $\mathbf{L'}$, $\mathbf{L''}$, we find that

$$\mathrm{rot}(\mathbf{H}') - \varepsilon \frac{d\mathbf{E}''}{dt} - \mathrm{grad}(\mathbf{K}') = 0 \tag{109}$$

$$-\mathrm{rot}(\mathbf{H}'') + \varepsilon \frac{d\mathbf{E}'}{dt} + \mathrm{grad}(\mathbf{K}'') = 0 \tag{110}$$

$$\mathrm{rot}(\mathbf{E}') + \mu \frac{d\mathbf{H}''}{dt} + \mathrm{grad}(\mathbf{L}') = 0 \tag{111}$$

$$\mathrm{rot}(\mathbf{E}'') + \mu \frac{d\mathbf{H}'}{dt} + \mathrm{grad}(\mathbf{L}'') = 0 \tag{112}$$

$$\mathrm{div}(\mathbf{E}') - \frac{\rho}{2\varepsilon} = 0,\ \mathrm{div}(\mathbf{H}') - \frac{\sigma}{2\mu} = 0 \tag{113}$$

$$\mathrm{div}(\mathbf{E}'') - \frac{\rho}{2\varepsilon} = 0,\ \mathrm{div}(\mathbf{H}'') - \frac{\sigma}{2\mu} = 0 \tag{114}$$

Due to the symmetry of equations (109)-(114) we have:

$$\mathbf{E}' = \mathbf{E}'', \mathbf{H}' = \mathbf{H}'', \mathbf{K}' = \mathbf{K}'', \mathbf{L}' = \mathbf{L}'' \tag{115}$$

Denote that

$$\begin{aligned} &\mathbf{E} = \mathbf{E}' + \mathbf{E}'', \qquad \mathbf{H} = \mathbf{H}' + \mathbf{H}'', \\ &\mathbf{K} = \mathbf{K}' + \mathbf{K}'', \qquad \mathbf{L} = \mathbf{L}' + \mathbf{L}'' \end{aligned} \tag{116}$$

Subtracting equation (110) from equation (109), we obtain that

$$\mathrm{rot}(\mathbf{H}) - \varepsilon \frac{d\mathbf{E}}{dt} - \mathrm{grad}(\mathbf{K}) = 0 \tag{117}$$

Similarly, subtracting equation (112) from (111), we also obtain that

$$\mathrm{rot}(\mathbf{E}) + \mu\frac{d\mathbf{H}}{dt} + \mathrm{grad}(\mathbf{L}) = 0 \tag{118}$$

Similarly, with equations (113) and (114), we can obtain the following formulae:

$$\mathrm{div}(\mathbf{E}) - \rho/\varepsilon = 0 \tag{119}$$
$$\mathrm{div}(\mathbf{H}) - \sigma/\mu = 0 \tag{120}$$

The resulting equations are necessary conditions for the existence of an extremum of functional (101) with respect to pairs of functions of the form $\mathbf{E}'$, $\mathbf{E}''$. These extrema are opposite (minimum-maximum or maximum-minimum) because the corresponding equations differ in the signs of the terms. Consequently, these equations are necessary conditions for the existence of a saddle line with respect to functions of the forms $\mathbf{E}'$ and $\mathbf{E}''$ in functional (101).

It can be seen that equations (116)-(120) are Maxwell's symmetric equations, where

E is the electric field strength,
H is the magnetic field strength,
μ is the magnetic permeability,
ε is the electric permittivity,
ρ is the electric charge density,
σ is the density of the hypothetical magnetic charge,
grad(**K**) is the electric current density,
grad(**L**) is the hypothetical magnetic current density.

Denote that

$$\mathbf{J} = \mathrm{grad}(\mathbf{K}) \tag{121}$$
$$\mathbf{M} = \mathrm{grad}(\mathbf{L}) \tag{122}$$

Let us consider the physical meaning of the quantity **K**. Denote that

ϕ is the electric scalar potential,
ϑ is the electrical conductivity,
j_x is the projection of the electric current density vector **J** onto the x-axis.

Then we get $j_x = -\vartheta d\boldsymbol{\phi}/dx$. But from (121) it follows that $j_x = d\mathbf{K}/dx$. Consequently,

$$\frac{d\mathbf{K}}{dx} = -\vartheta\frac{d\boldsymbol{\phi}}{dx} \tag{123}$$

i.e.

$$\mathbf{K} = -\vartheta\boldsymbol{\phi} \tag{124}$$

Likewise,

$$\frac{d\mathbf{L}}{dx} = -\varsigma\frac{d\boldsymbol{\varphi}}{dx} \tag{125}$$
$$\mathbf{L} = -\varsigma\boldsymbol{\varphi} \tag{126}$$

where

φ is the magnetic scalar potential,
ς is the magnetic conductivity.

So, combining equations (117), (118), (121), (122), we obtain the final form of Maxwell's equations:

$$\mathrm{rot}(\mathbf{H}) - \varepsilon\frac{d\mathbf{E}}{dt} - \mathbf{J} = 0 \tag{127}$$
$$\mathrm{rot}(\mathbf{E}) + \mu\frac{d\mathbf{H}}{dt} + \mathbf{M} = 0 \tag{128}$$
$$\mathrm{div}(\mathbf{E}) - \rho/\varepsilon = 0 \tag{129}$$
$$\mathrm{div}(\mathbf{H}) - \sigma/\mu = 0 \tag{130}$$

Thus, a functional is obtained for which the Maxwell equations are necessary conditions for the existence of a saddle line. It is also proved in (Khmelnik, 2014) that these equations are also sufficient conditions for the existence of a saddle line. This functional takes into account heat losses and the existence of magnetic charges and currents.

The author calls the saddle line search principle used in the variational extremum principle and shows that it is applicable in various areas of physics. It can be assumed that the variational principle is not just a beautiful record of patterns found in a different way (like a rotor that combines three linear equations), but the original information from which these patterns follow. Then it is possible to look for new patterns based on the variational principle. This is the method we will use next.

Direct current and Lorentz force

From Maxwell's equations of general form (1)-(4) it follows that for a static field there should be the following set of equations:

$$\mathrm{rot}(\mathbf{E}) = 0 \tag{131}$$
$$\mathrm{rot}(\mathbf{H}) = \mathbf{J} \tag{132}$$
$$\mathrm{div}(\mathbf{E}) = 0 \tag{133}$$
$$\mathrm{div}(\mathbf{H}) = 0 \tag{134}$$

These four equations can be obtained by discarding time derivatives. This set of equations follows from (1)-(4) simply because the set of equations (1)-(4) is, in turn, a consequence of the variational principle.

In the DC wire we have:

$$\mathbf{E} = \rho\mathbf{J} \tag{135}$$

where ρ is the resistivity of the wire. Therefore, there is a field in the DC wire described by a set of equations of the following form:

$$\mathrm{rot}(\mathbf{J}) = 0 \tag{136}$$
$$\mathrm{rot}(\mathbf{H}) = \mathbf{J} \tag{137}$$
$$\mathrm{div}(\mathbf{J}) = 0 \tag{138}$$
$$\mathrm{div}(\mathbf{H}) = 0 \tag{139}$$

Usually, the set of Maxwell equations and the Lorentz force formula are considered as the foundations of electrodynamics, as two independent components of these foundations. It must be said that Maxwell actually included the Lorentz force in one of his equations in the following form (Maxwell, 1873a, 1873b):

$$\mathbf{J} = \sigma\left(-\nabla\boldsymbol{\varphi} - \frac{\partial\mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B}\right) \tag{140}$$

where **J**, **v**, **B**, **φ**, **A** are the current, speed of movement of an electric charge, magnetic induction, electric and magnetic potentials, respectively. Further, Maxwell's equations were transformed by the work of Heaviside, Hertz and Gibbs into a modern form, where there is neither the speed of the electric charge nor the potentials. In this case, the formula for the Lorentz force is:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \tag{141}$$

or, taking into account equation (135),

$$\mathbf{F} = \rho q\mathbf{J} + \mathbf{J}\times\mathbf{B} \tag{142}$$

where q is an electric charge, complements the set of Maxwell's equations. However, these forces act inside the physical system, which is described by the set of Maxwell equations with an additional equation (142). In a closed physical system, this entire group of formulas must be consistent.

A wire with a direct current (DC) is a testing ground for this statement. Since the set of Maxwell's equations is strictly defined, formula (142) must follow from this equations' set. Thus, Lorentz force formula (142) should follow from the solution of Maxwell's equations (136) and (139) for a DC wire.

This solution was found in the cylindrical coordinate system $\{r, \varphi, z\}$ and has the following form (Khmelnik, 2021a):

$$J_\varphi = j_\varphi \mathrm{co} + J_{\varphi o} \tag{143}$$
$$J_z = j_z \mathrm{co} + J_{zo} \tag{144}$$
$$H_r = h_r \mathrm{co} + H_{ro} \tag{145}$$
$$H_\varphi = h_\varphi \mathrm{si} + H_{\varphi o} \tag{146}$$
$$H_z = h_z \mathrm{si} + H_{zo} \tag{147}$$
$$\mathrm{co} = \cos(\alpha\varphi + \chi z) \tag{148}$$
$$\mathrm{si} = \sin(\alpha\varphi + \chi z) \tag{149}$$

where α and χ are some constants, $j_z(r)$, $h_z(r)$, $J_{zo}(r)$, $H_{zo}(r)$, etc. are some functions of the coordinate r.

With these known functions (143)-(147), energy flows in a DC wire can be determined. The density of the electromagnetic energy flux, namely the Poynting vector is determined, as it is known, by the following formula:

$$\mathbf{S} = \mathbf{E}\times\mathbf{H} \tag{150}$$

The currents correspond to the electric field strengths of the same designation in equations (5). Combining formulae (150) and (135), we get:

$$\mathbf{S} = \rho\mathbf{J}\times\mathbf{H} = \frac{\rho}{\mu}\mathbf{J}\times\mathbf{B} \tag{151}$$

The Lorentz magnetic force acting on a unit charge of a conductor in a unit volume is the volumetric density of the Lorentz magnetic force equal to

$$\mathbf{F_m} = \mathbf{J}\times\mathbf{B} \tag{152}$$

From equations (151) and (152) we find:

$$\mathbf{F_m} = \mu\,\mathbf{S}/\rho \tag{153}$$

Therefore, <u>in a DC wire, the density of the Lorentz magnetic force is proportional to the Poynting vector</u>. Lorentz electric force follows from formula (135). So, we have

$$\mathbf{F_e} = \rho q\mathbf{J} \tag{154}$$

Consequently, the solution of the set of Maxwell's equations allows one to find the energy flux density from equation (150) and then find the volume density of the Lorentz force from equation (142). Thus, the <u>formula for the Lorentz force is a consequence of the set of Maxwell's equations, and not an addition to this set</u>.

Structure of an electromagnetic wave

So, the wave equation of electrodynamics, as the only and really existing solution of Maxwell's equations, is a myth far from reality. Nature is much more diverse. For instance, here there is the photograph of a wire wetted with a magnetic fluid, where the reader can see the spiral lines formed by the fluid that is shown in Figure 3.

Fig. 3. The photograph of a wire wetted with a magnetic fluid.

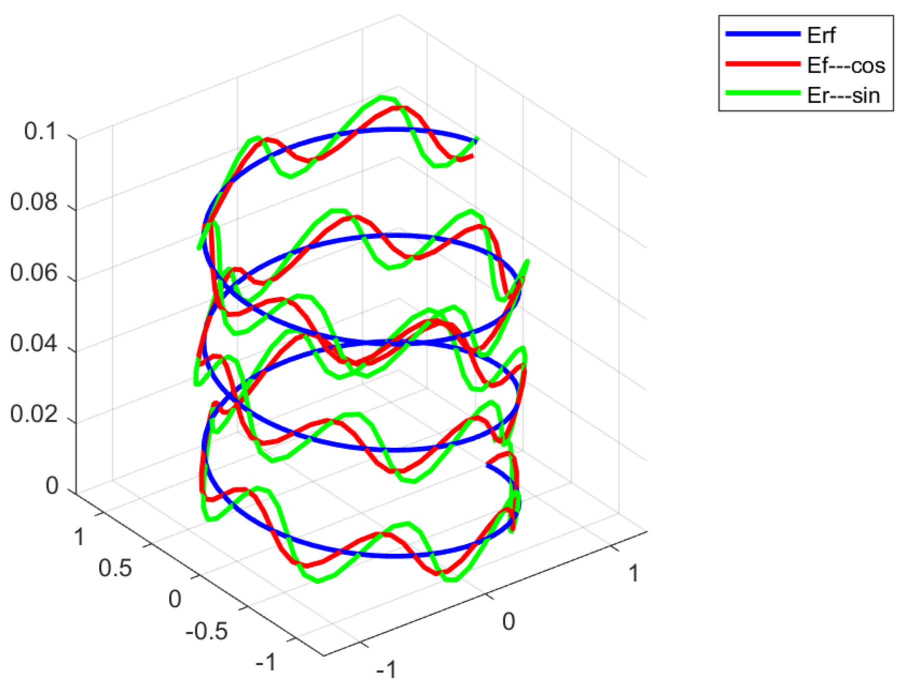

Fig. 4. The spirals.

Spirals are present in all solutions of Maxwell's equations discussed above. For instance, Figure 4 shows some spirals corresponding to the following functions:

$$\overrightarrow{\mathbf{E}_{\varphi}}, \quad \overrightarrow{\mathbf{E}_{\mathrm{r}}}, \quad \overrightarrow{\mathbf{E}_{\mathrm{r}\varphi}} = \overrightarrow{\mathbf{E}_{\varphi}} + \overrightarrow{\mathbf{E}_{\mathrm{r}}}$$

However, there are phenomena more mysterious. For instance, here there are two photographs (Figures 5 and 6) of the square waves in which large ships die.

Solutions of Maxwell's equations can also have a similar form. The strengths of the electric and magnetic fields found as a solution to Maxwell's equations can have the following form (Khmelnik, 2021b):

$$E_x(x,y,z,t) = e_x \cos(\alpha x)\sin(\alpha y)\sin(\alpha z)\sin(\omega t) \qquad (155)$$

$$E_y(x,y,z,t) = e_y \sin(\alpha x)\cos(\alpha y)\sin(\alpha z)\sin(\omega t) \qquad (156)$$

$$E_z(x,y,z,t) = e_z \sin(\alpha x)\sin(\alpha y)\cos(\alpha z)\sin(\omega t) \qquad (157)$$

$$H_x(x,y,z,t) = h_x \sin(\alpha x)\cos(\alpha y)\cos(\alpha z)\cos(\omega t) \quad (158)$$

$$H_y(x,y,z,t) = h_y \cos(\alpha x)\sin(\alpha y)\cos(\alpha z)\cos(\omega t) \quad (159)$$

$$H_z(x,y,z,t) = h_z \cos(\alpha x)\cos(\alpha y)\sin(\alpha z)\cos(\omega t) \quad (160)$$

where e_x, e_y, e_z, h_x, h_y, h_z are constant function amplitudes; α, ω are constants. The amplitudes in it are related by equations of the following form:

$$h_z = 0 \quad (161)$$

$$h_y = -h_x \quad (162)$$

$$h_x = -\frac{\varepsilon\omega}{\alpha} e_x \quad (163)$$

$$e_y = e_x \quad (164)$$

$$e_z = -2e_x \quad (165)$$

The amplitudes can be determined by some given value of

Fig. 5. The structure of the ocean square waves.

Fig. 6. The other picture of the ocean square waves.

the parameter e_x. The circular frequency is

$$\omega = c\alpha\sqrt{4.5} \tag{166}$$

These equations describe a volumetric standing wave that exists in the volume of a cube whose edge has the following length:

$$L = \pi/\alpha \tag{167}$$

The electromagnetic energy density of this wave is defined as

$$W = \varepsilon E^2 + \mu H^2 \tag{168}$$

and this wave satisfies the following condition:

$$U = \varepsilon|E^2| = \mu|H^2| \tag{169}$$

Total electromagnetic wave energy in the cube is

$$W_o = U \cdot L^3 \tag{170}$$

This energy does NOT change with time.

Epilogue

"Have the reader ever heard that Maxwell's equations, the wave equation, the vector potential underlie quantum mechanics – a fundamental physical theory that ..." Quiet, guys, the author is not arguing with the reader. The fact that even the incorrect solution of Maxwell's equations came turned out to be necessary increases my admiration for these equations. The law of conservation of energy is not written for the reader. Otherwise, the reader would not have written so ornately: "*The energy of a particle remains a quantity invariant with respect to the translation of time.*" Therefore, the reader can use what the reader has underlie. However, the author urges classical electrodynamics to be like all classical sciences and not to break down in front of young people. We'll have to see what she'll grow up to be when she's wiser.

CONCLUSION

This paper had the aim to stimulate further researches in this direction that can be followed young researchers. Some possible solutions of the Maxwell equations were demonstrated and visualized. The phenomenon of the ocean square waves was also touched and discussed.

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